Comparing the two results we see that the cone empties in $\frac{8}{15}$ of the time it takes the cylinder, or the cylinder takes $1\frac{7}{8}$ as long as the cone to empty. The minus sign is prefixed because x decreases as t increases.

Also solved by *G. B. M. ZERR*, *ELMER SCHUYLER*, *J. SCHEFFER*, and *J. C. NAGLE*.

NOTE. In reference to problem 63, Dr. Arnold Emch says: "I had the problem solved by my class in graphic statics by a purely graphical method and the following values (approximations) were obtained: $\angle ABE=46°$, $\angle BAD=56°$, tension in $BE=56.8$, tension in $AD=71$. This shows that the solution in the MONTHLY is correct."

DIOPHANTINE ANALYSIS.

71. Proposed by A. H. BELL, Hillsboro, Ill.

Find five numbers such that the product of any two plus 1 will equal a square.

III. Solution by M. A. GRUBER, A. M., War Department, Washington. D. C.

By using $(s-1)^2$, the denominator of Euler's fifth number, where $s=4n(n-1)(n+1)[4n(2n-1)(2n+1)]$, I have found five numbers in terms of n: $x=n-1$, $y=n+1$, $z=4n$, $w=4n(2n-1)(2n+1)$, and

$$v=\frac{4n(2n-1)(2n+1)[2n(2n-1)-1][2n(2n+1)-1](8n^2-1)}{\{4n(n-1)(n+1)[4n(2n-1)(2n+1)]-1\}^2}.$$

The numerator of v is four times the product of the roots of the six squares $xy+1$, $xz+1$, $yz+1$, $xw+1$, $yw+1$, and $zw+1$.

The denominator of v is the square of $(xyzw-1)$.

Take $n=1, 2, 3, 4, 5, 6$, etc. We then obtain the following sets of five numbers:

$$0,\ 2,\ 4,\ 12,\ 420;$$

$$1,\ 3,\ 8,\ 120,\ \frac{777480}{(2879)^2};$$

$$2,\ 4,\ 12,\ 420,\ \frac{35455980}{(40319)^2};$$

$$3,\ 5,\ 16,\ 1008,\ \frac{499902480}{(241919)^2};$$

$$4,\ 6,\ 20,\ 1980,\ \frac{3822388020}{(950399)^2};$$

$$5,\ 7,\ 24,\ 3432,\ \frac{20000100120}{(2882879)^2};\ \text{etc., etc., etc.}$$

We also find

$$xv+1=\frac{\{(2n-1)(2n+1)[2n(2n-1)-1][2n(2n+1)-1]-2n(8n^2-1)\}^2}{(s-1)^2};$$

$$yv+1=\frac{\{(2n-1)(2n+1)[2n(2n-1)-1][2n(2n+1)-1]+2n(8n^2-1)\}^2}{(s-1)^2};$$

$$zv+1=\frac{\{16n^4(2n-1)(2n+1)-(8n^2-1)\}^2}{(s-1)^2};\text{ and}$$

$$wv+1=\frac{\{24n^2(2n^2-1)(2n-1)(2n+1)-1\}^2}{(s-1)^2}.$$

74. Proposed by O. W. ANTHONY, M. Sc., Instructor in Mathematics, Boys' High School, New York City.

Solve $x^2+y^2=\square$, $z^2+w^2=\square$, $y^2+w^2=\square$.

I. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

Take any two integral equations in which the sum of two squares equals a square, as

$$a^2+b^2=c^2, \text{ and } a_1^2+b_1^2=c_1^2.$$

Multiply the terms of the first equation by the first term and the second term, respectively, of the second equation. Also multiply the terms of the second equation by the first term and the second term, respectively, of the first equation. We then have

$$(aa_1)^2+(a_1b)^2=(a_1c)^2 \ldots .(1),$$
$$(ab_1)^2+(bb_1)^2=(b_1c)^2 \ldots .(2),$$
$$(aa_1)^2+(ab_1)^2=(ac_1)^2 \ldots .(3),$$
$$(a_1b)^2+(bb_1)^2=(bc_1)^2 \ldots (4).$$

Now put $x=aa_1$, $y=a_1b$, $z=ab_1$, and $w=bb_1$; then equations (1), (2), and (4) are the three required by the problem, there being added, in the solution, $x^2+z^2=\square \ldots .(3)$.

By means of the formula $(2mn)^2+(m^2-n^2)^2=(m^2+n^2)^2$, find a few integral numerical equations.

Take $m=2$, $n=1$; then $4^2+3^2=5^2 \ldots .(1)$.

Take $m=3$, $n=2$; then $12^2+5^2=13^2 \ldots .(2)$.

Take $m=4$, $n=1$; then $8^2+15^2=17^2 \ldots .(3)$.

Take $m=5$, $n=2$; then $20^2+21^2=29^2 \cdots\cdot(4)$, etc.

From (1) and (2), $x=48$, $y=36$, $z=20$, $w=15$.

From (1) and (3), $x=32$, $y=24$, $z=60$, $w=45$.

From (1) and (4), $x=80$, $y=60$, $z=84$, $w=63$.

From (2) and (3), $x=96$, $y=40$, $z=180$, $w=75$, etc.